Encoded Universality from a Single Physical Interaction

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Outline

- Background & Motivation
- Exchange Hamiltonian
- Encoded Universality from Isotropic Exchange Hamiltonian
- Encoded Universality from Anisotropic Exchange Hamiltonian
- Error Correction and Leakage
- Decoherence-free Subspaces
- Open Problems

Traditional Approach to Universal Computation

- Generate Universal Gate Set

  Example: Hadamard, phase, $\pi/8$ and C-NOT

- Find physical implementation for set

Problem: Don’t always have a reliable physical implementation

Basic Idea of “Encoded Universality”

Let L be quantum gate library that is not universal

Encoding qubits in larger Hilbert space & applying L may be universal in original space

Hamiltonians and Unitary Operators

Time evolution of quantum state described by Schrödinger Equation

$$i\hbar\frac{d\left|\psi\right\rangle}{dt} = H\left|\psi\right\rangle$$

Gives unitary operator

$$U\left(t_1,\ t_2\right) = \exp\left[\frac{-iH\left(t_2 - t_1\right)}{\hbar}\right]$$

Given a set of primitive Hamiltonians what others can be obtained?

Trotter and Baker-Campbell-Hausdorf Formulae

Rules for combining Hamiltonians:

$$e^{i(\alpha \mathbf{A}+\beta \mathbf{B})} = \lim_{p\to\infty} \left(e^{i\alpha\mathbf{A}/p} \cdot e^{i\beta\mathbf{B}/p} \right)^p$$

$$e^{i[\mathbf{A},\mathbf{B}]} = \lim_{p\to\infty} \left(e^{-i\mathbf{A}\sqrt{p}} \cdot e^{i\mathbf{B}/\sqrt{p}} \cdot e^{i\mathbf{A}/\sqrt{p}} \cdot e^{-i\mathbf{B}\sqrt{p}} \right)^p$$

$\Rightarrow$ New Hamiltonians formed by linear combinations

$$\alpha\mathbf{A} + \beta\mathbf{B}$$

and Lie-commutators

$$i\,[\mathbf{A},\mathbf{B}] = i\,(\mathbf{AB} - \mathbf{BA})$$

Heisenberg Exchange Hamiltonian

$$H_{ij} = J_{ij}^X \sigma_x^i \sigma_x^j + J_{ij}^Y \sigma_y^i \sigma_y^j + J_{ij}^Z \sigma_z^i \sigma_z^j$$

Isotropic Case:

$$H_{ij} = J_{ij} \sum_{i \neq j} \left(\sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j + \sigma_z^i \sigma_z^j \right)$$

Example:

$$H_{12} = J_{ij} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Setting $J_{ij} = 1$

$$\frac{1}{2}(I + H_{12}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Encoded Universality

Isotropic exchange interaction not universal

However it is universal in a subspace

Use representation theoretic analysis to find subspace and encoding
Encoding efficiency $\rightarrow 1$ as $n \rightarrow \infty$

Must also introduce tensor structure
conjoining encoded qubits

Explicit Encoding

$$
\begin{aligned}
|0_L\rangle &= \frac{1}{\sqrt{2}}\left(|010\rangle - |100\rangle\right) \\
|1_L\rangle &= \sqrt{\frac{2}{3}}|001\rangle - \sqrt{\frac{1}{6}}|010\rangle - \sqrt{\frac{1}{6}}|100\rangle
\end{aligned}
$$

Single-qubit gates implemented with ≤ 4 exchange interactions

Nontrivial two-qubit gate implemented with 19 exchange interactions

Anisotropic Exchange Hamiltonian (XY-interaction)

$$H_{ij} = \frac{J_{ij}}{2}\left(\sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j\right)$$

Relevant for quantum dot spins & cavity QED

Can achieve encoded universality for qutrit with 3 physical qubits

Explicit encoding:

$$|0_L\rangle = |100\rangle \quad |1_L\rangle = |010\rangle \quad |2_L\rangle = |001\rangle$$

Universality Criterion

Not every physical interaction gives encoded universality

Generally must determine on case by case basis

Necessary Condition:

# of linearly independent operators in Lie algebra of H_n must $>$ poly(n)

Example:

$$\left\{\sigma_z^i,\ \sigma_x^i\sigma_x^{i+1}\right\}$$

has no universal encoding

Error Correction and Leakage

Standard FT procedures apply

Can be concatenated within stabilizer code

“Leakage” can be a problem

Authors give procedure for dealing with leakage

Decoherence-Free Subspaces

Qubits encoded in subspace invariant to collective decoherence

Example:

Collective Dephasing $(|0\rangle \to |0\rangle,\ |1\rangle \to e^{i\alpha}|1\rangle)$

Encode Basis States $|1\rangle \to |10\rangle,\ |0\rangle \to |01\rangle$

Open Questions/Problem

- What other interactions yield “encoded universality”?
- Affect of additional restriction on universality
  (e.g. only nearest neighbor interactions allowed)
- (Optimal) synthesis for encoded gates