Quantum Circuits Seminar, Sept. 16 2003

Tutorial: Basic Concepts in Quantum Circuits

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Outline

- Motivation
- Quantum vs. Classical
- Quantum Gates
- Quantum Circuits
- Physical Implementation

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- **Motivation**
- Quantum vs. Classical
- Quantum Gates
- Quantum Circuits
- Physical Implementation

Computational Limits

- Some important computational problems seem to be permanently intractable
  - Their complexity grows exponentially with problem size, e.g. factoring large numbers—the basis for “unbreakable” Internet codes
- Performance improvements in “classical” computer circuits may be approaching a limit
  - This is described by Moore’s Law

Computational Limits

- **Question:** Is there a faster and more compact way to compute?
- **Answer:** Yes !

  Quantum mechanics can form the basis for an entirely new type of computation— quantum computing — if some huge practical implementation problems can be solved

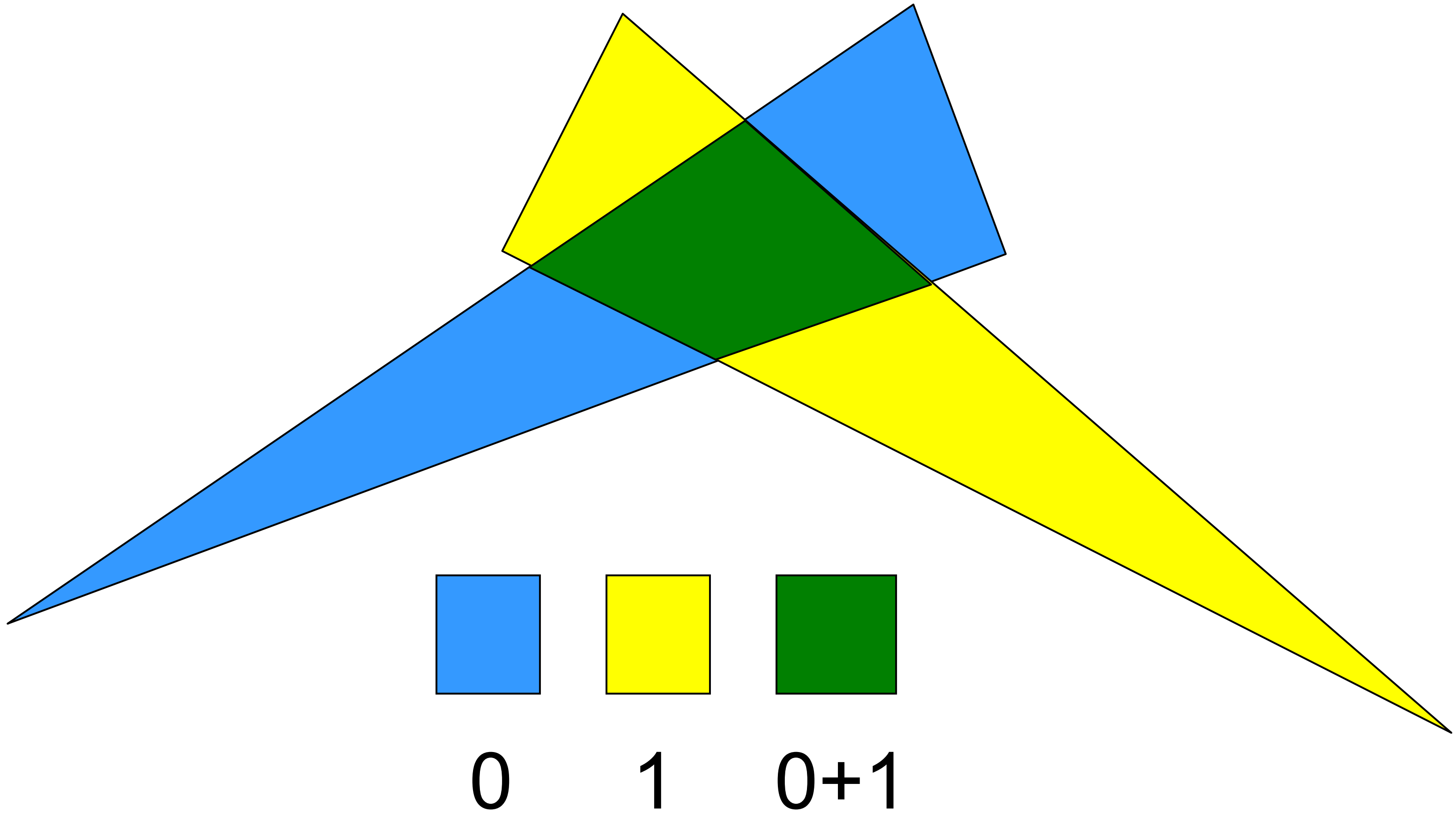
Quantum Information

- A classical logic state can be 0 or 1, but not both
- A quantum state *can* be 0 and 1 at the same time!
- More precisely, a quantum state is a superposition of the zero and one states called a **qubit**

$$c_0|0\rangle + c_1|1\rangle$$

The coefficients c_0 and c_1 are complex numbers called (probability) amplitudes

Quantum Information



Quantum Information

- **The Good News**
  - N qubits can store 2^N binary numbers simultaneously, suggesting massive parallelism

    $N = 2$: $|\Psi\rangle = c_0|00\rangle + c_1|01\rangle + c_2|10\rangle + c_3|11\rangle$

    or, in general,

$$|\Psi\rangle = \sum_{i=0}^{2^n-1} c_i \left| b_{i,n-1} b_{i,n-2} \ldots b_{i,0} \right\rangle$$

  - Quantum states have wavelike properties that allow powerful nonclassical operations (interference, entanglement)

Quantum Information

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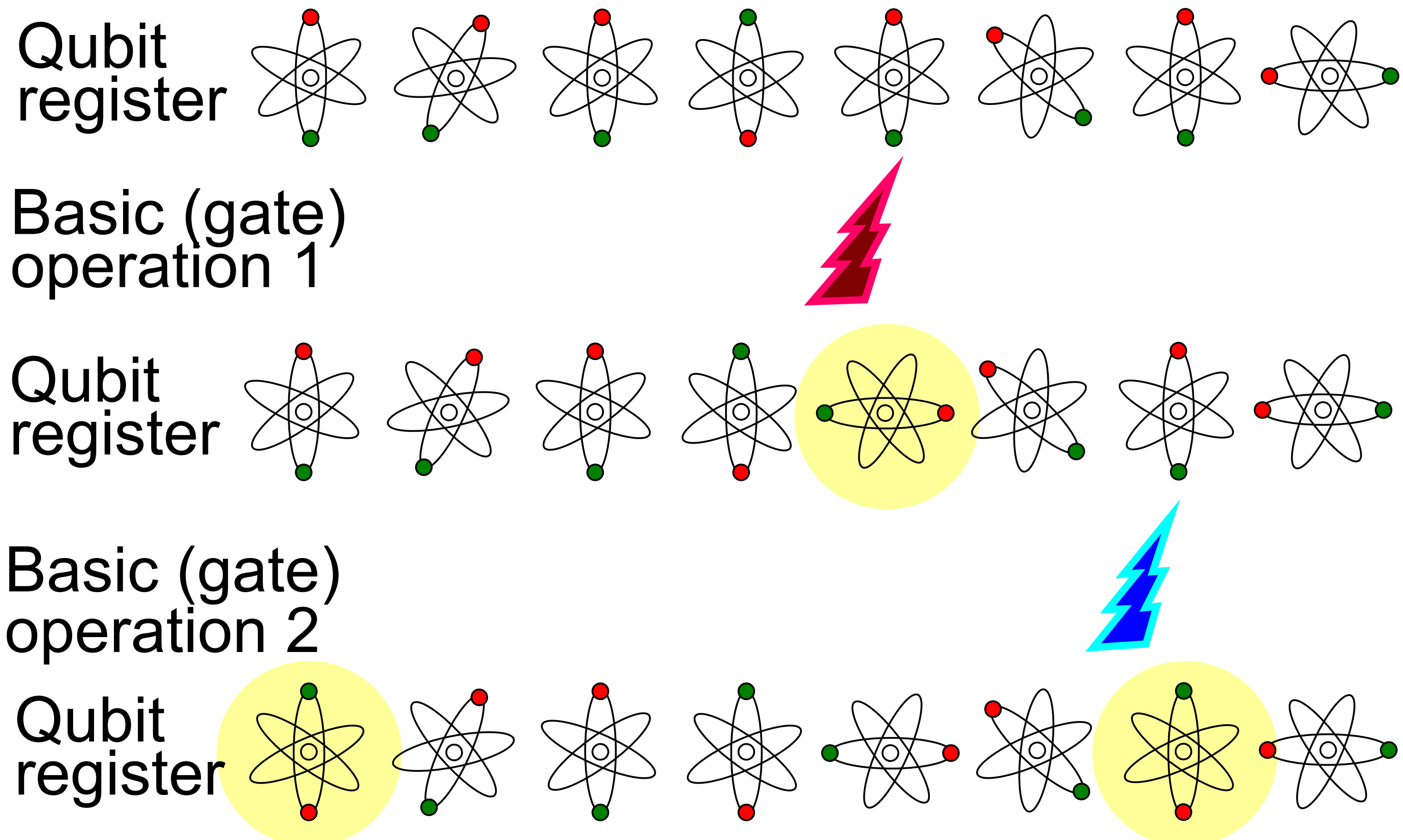
  - > Quantum states have wavelike properties that allow powerful nonclassical operations (interference, entanglement)

Quantum Information

- **The Bad News**
  - Measurement yields just one of the 2^N superimposed numbers $|b_{i,n-1}\, b_{i,\, n-2} \ldots b_{i,0}\rangle$ and destroys the superposition
  - Quantum states are very fragile due to
    - Tiny (nano) scale and low energy levels
    - Interaction with the environment (decoherence)
- **Implications**
  - Physical quantum circuits are extremely hard to build
  - Fault-tolerant design is believed to be essential

Quantum Computing



A Little History

- **1982**: Richard Feynman suggested quantum mechanics could provide an exponential speed-up in simulation
- **1985**: David Deutsch described a simple algorithm exhibiting quantum parallelism
- **1994**: Peter Shor showed how to factor integers into primes in polynomial time using quantum methods, thus “breaking” RSA encryption
- **1996-now**: First quantum computing devices built at LANL, Oxford, etc. employing a few (≤ 10) qubits

Outline

- Motivation
- **Quantum vs. Classical**
- Quantum Gates
- Quantum Circuits
- The Future

Classical Logic Circuits

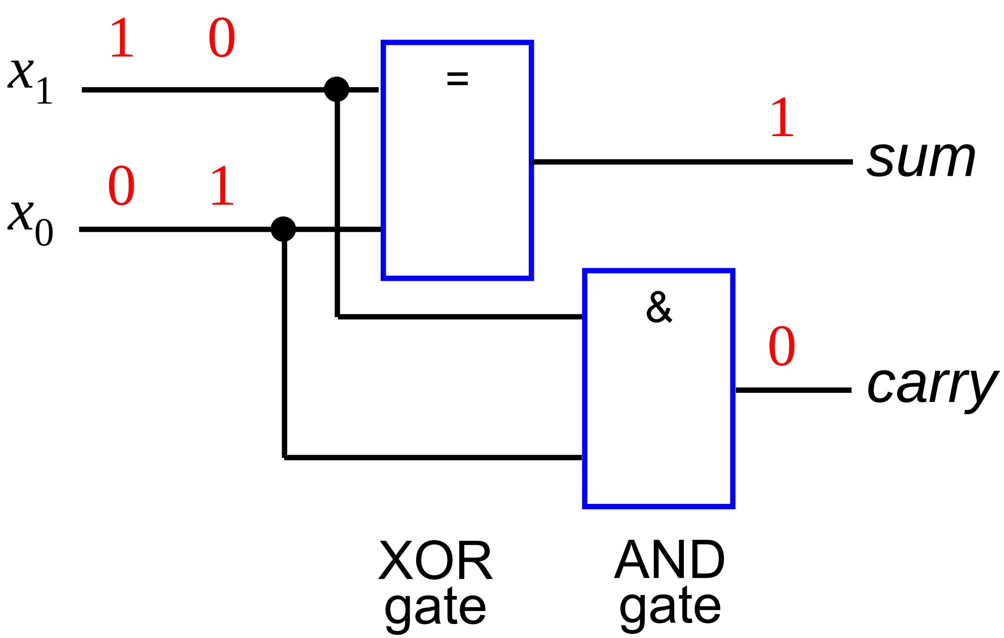
- Behavior is governed implicitly by classical physics: no restrictions on copying or measuring signals
- Signal states are simple bit vectors, e.g. X = 01010111
- Signal operations are defined by Boolean algebra
- Small well-defined sets of universal gate types exist , e.g. {NAND}, {AND, OR, NOT}
- Circuits use fast, scalable and macroscopic technologies such as transistor-based CMOS integrated circuits

Quantum Circuits

- Behavior is governed by quantum mechanics
- Signal states are qubit vectors
- Operations are defined by linear algebra over Hilbert space and represented by unitary matrices
  - Gates and circuits must be reversible (information-lossless)
  - Number of output lines = Number of input lines
  - States cannot be copied so fan-out ("cloning") is not allowed
- Many universal gate sets and physical implementation technologies exist (the best ones are not obvious)

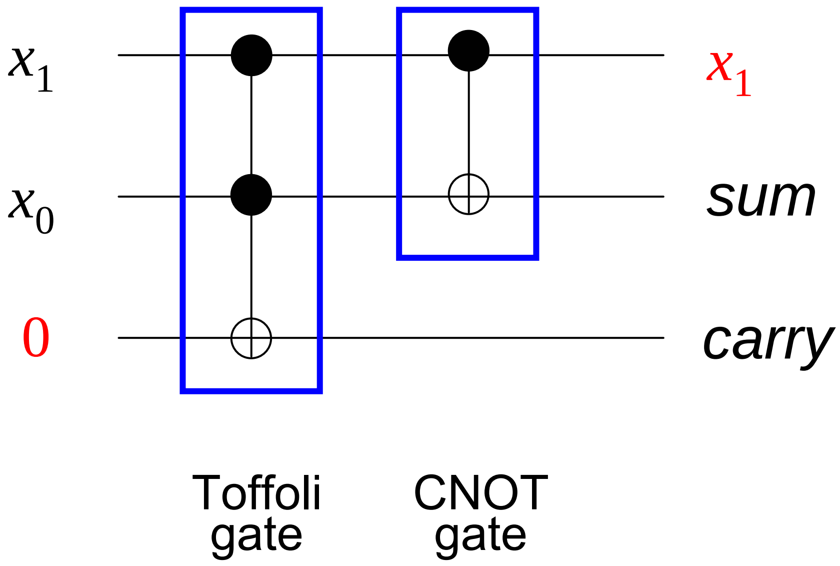
Classical vs. Quantum Circuits

- **Example: Classical Half Adder**
  - Compute the sum and carry for two bits x_1,x_0



Classical vs. Quantum Circuits

- **Example: Quantum Half Adder**
  - Compute the sum and carry for two qubits x_1, x_0



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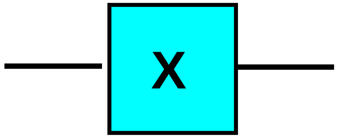
Quantum Gates

- **One-Input gate:** NOT
  - Input state: $c_0|0\rangle + c_1|1\rangle$
  - Output state: $c_1|0\rangle + c_0|1\rangle$
  - Graphic symbol:



  - Basic states $|0\rangle$ and $|1\rangle$ are mapped thus:

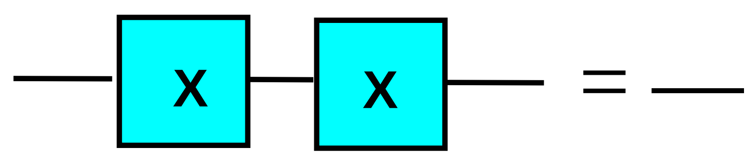
$$|0\rangle \rightarrow |1\rangle$$

$$|1\rangle \rightarrow |0\rangle$$

Quantum Gates

- **NOT gate** (contd.)
  - > Vector notation for states: $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
  - > Matrix notation for gate operation: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
  - > Gate connection corresponds to matrix multiplication:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



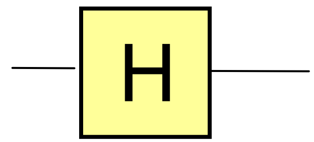
Identity matrix

NOT matrix

Quantum Gates

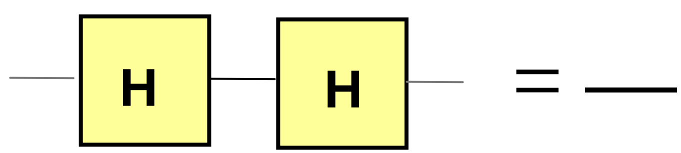
- **Hadamard Gate**

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$



> Maps $|0\rangle \rightarrow 1/\sqrt{2}\,|0\rangle + 1/\sqrt{2}\,|1\rangle$ and $|1\rangle \rightarrow 1/\sqrt{2}\,|0\rangle - 1/\sqrt{2}\,|1\rangle$ so it “randomizes” the basic states

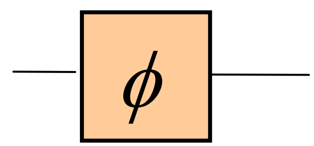
$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$



Quantum Gates

- **Phase-Shift Gate**

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$



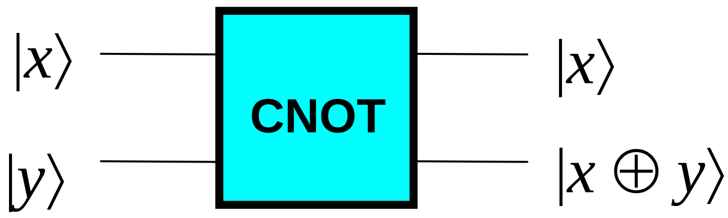
> Maps $|0\rangle \rightarrow |0\rangle$ and $|1\rangle \rightarrow e^{i\phi}|1\rangle$ so it "twists" the 1 state by an angle ϕ

> If = π, it maps $|1\rangle \rightarrow -|1\rangle$

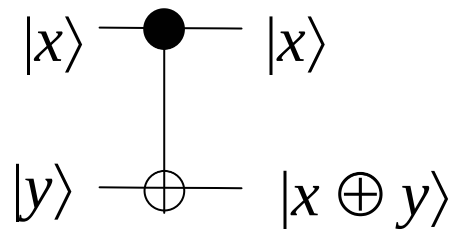
> Note that the entries of a gate matrix can be complex numbers

Quantum Gates

- **Two-Input Gate:** Controlled NOT (CNOT)



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



> CNOT maps

$$|x\rangle|0\rangle \rightarrow |x\rangle||x\rangle$$

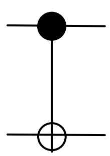
and

$$|x\rangle|1\rangle \rightarrow |x\rangle||\text{NOT}(x)\rangle$$

Quantum Gates

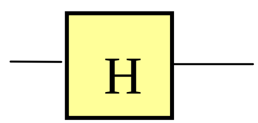
“Standard” Universal Gate Set

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



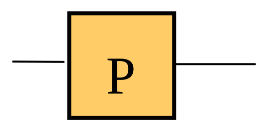
CNOT

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



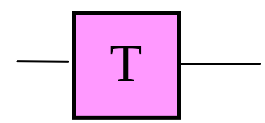
Hadamard

$$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$



Phase

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$



T (π/8) gate

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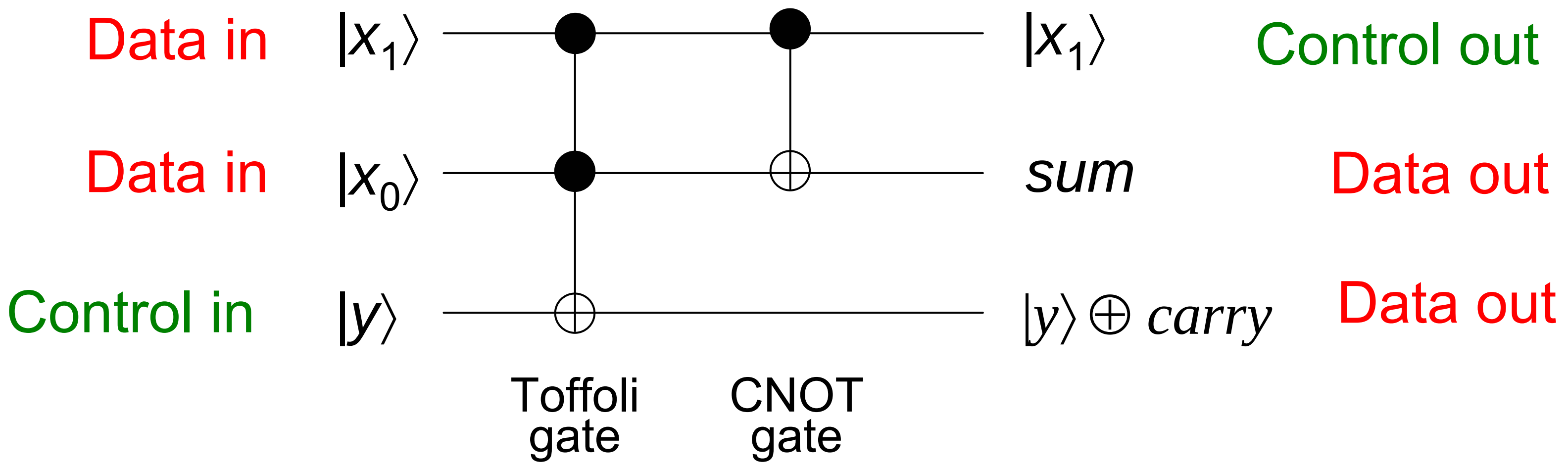
Quantum Circuits

- A quantum “circuit” is a sequence of quantum “gates”
- The signals (qubits) may be static while the gates are dynamic
- The circuit has fixed “width” corresponding to the number of qubits being processed
- Logic design (classical and quantum) attempts to find circuit structures for needed operations that are
  - Functionally correct
  - Independent of physical technology
  - Low-cost, e.g. uses the minimum number of qubits or gates

Quantum Circuits

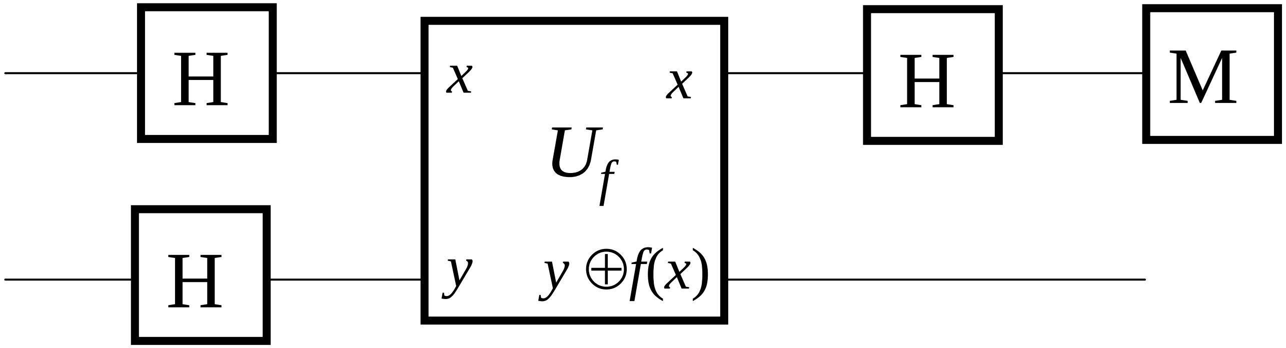
- **Example 1: Quantum Half Adder**
  - Compute the sum and carry for two qubits x_1,x_0



Quantum Circuits

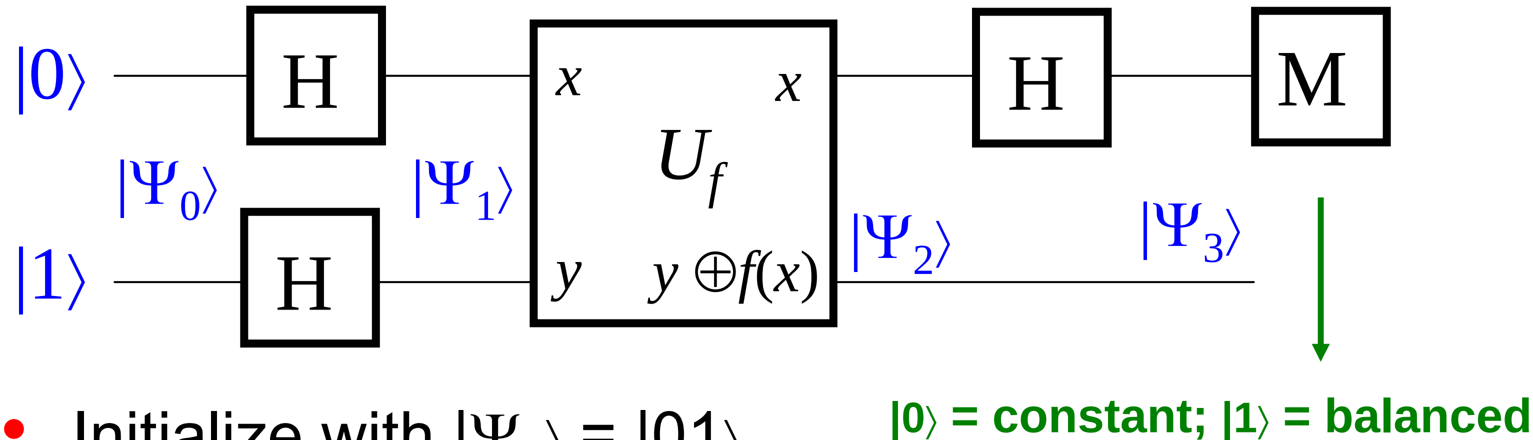
Example 2: Implementing Deutsch's Algorithm

- *Problem:* Determine whether a one-variable Boolean function $f(x)$ is constant, i.e. $f(0)= f(1)$, or balanced, i.e. $f(0) \neq f(1)$.
- Classical algorithms require two evaluations of f.
- This algorithm uses just one quantum evaluation by, in effect, computing $f(0)$ and $f(1)$simultaneously
- **Circuit**:



Quantum Circuits

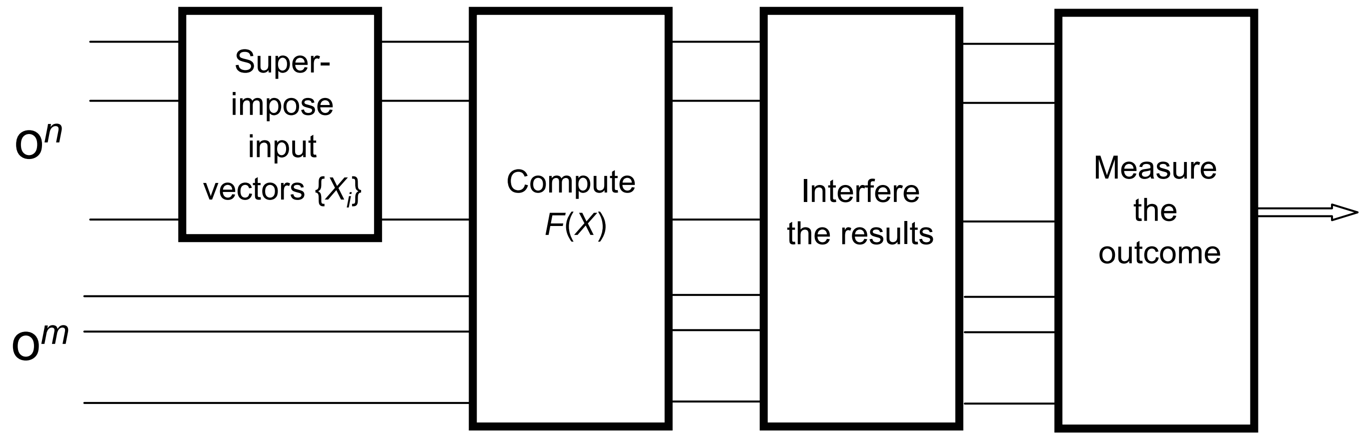
- **Deutsch's Algorithm** (contd.)



- Initialize with $|\Psi_0\rangle = |01\rangle$
- Create superposition of *x* states using the first Hadamard (H) gate. Set *y* control input using the second H gate
- Compute *f*(*x*) using the special unitary circuit U_f
- Interfere the $|\Psi_2\rangle$ states using the third H gate
- Measure the *x* qubit

Quantum Computation

- Generic Structure to Compute *F*(*X*)



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Physical Implementation

Main Contenders

- Nuclear magnetic resonance (NMR)
- Ion traps
- Semiconductor quantum dots
- Optical lattices

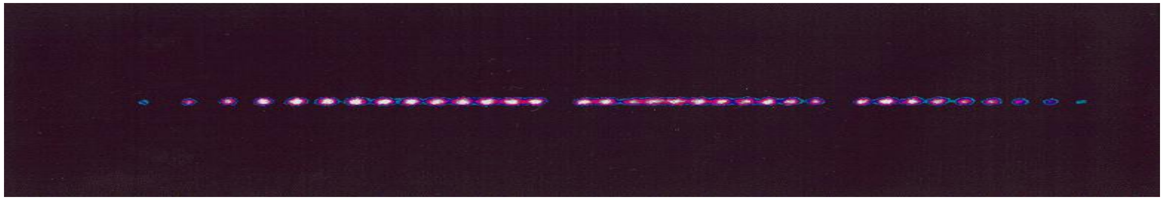
  etc.

Main Deficiency

- Poor scalability

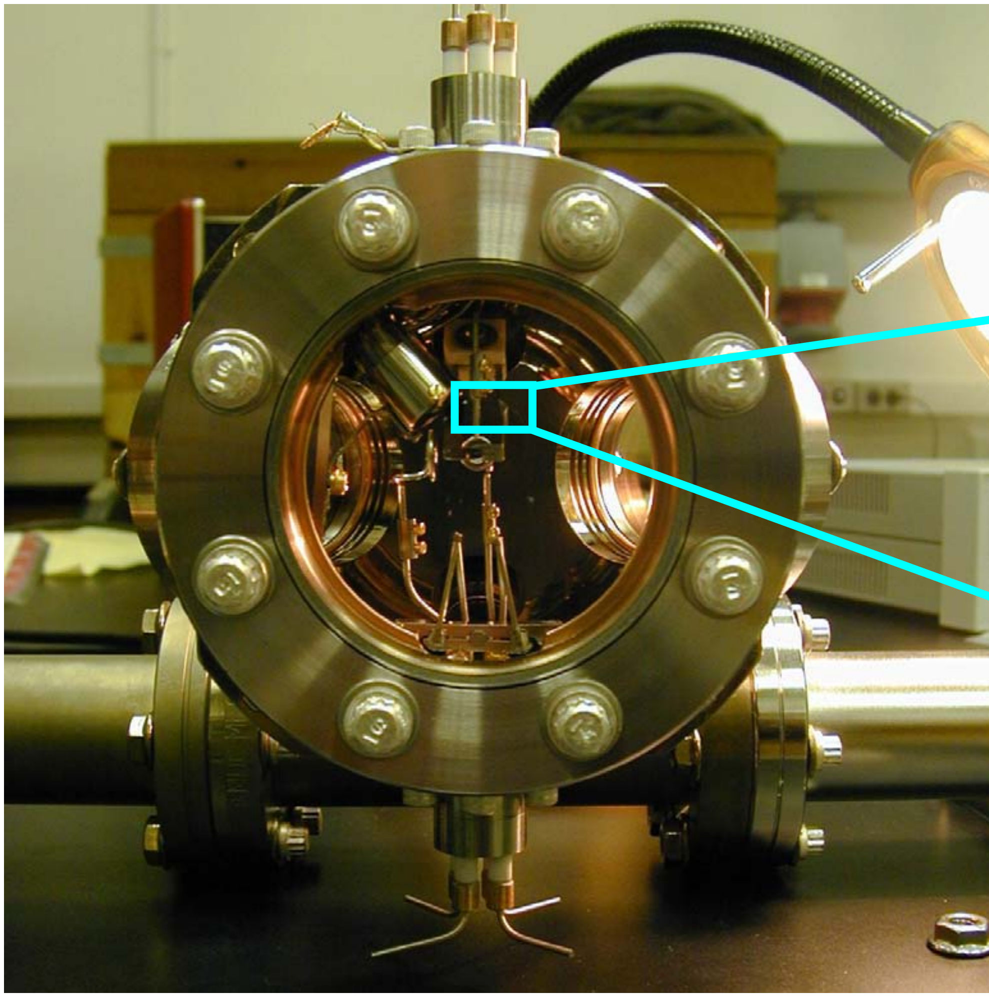
Ion Traps

- String of charged particles is trapped by a combination of static and oscillating electric fields in a high-vacuum device

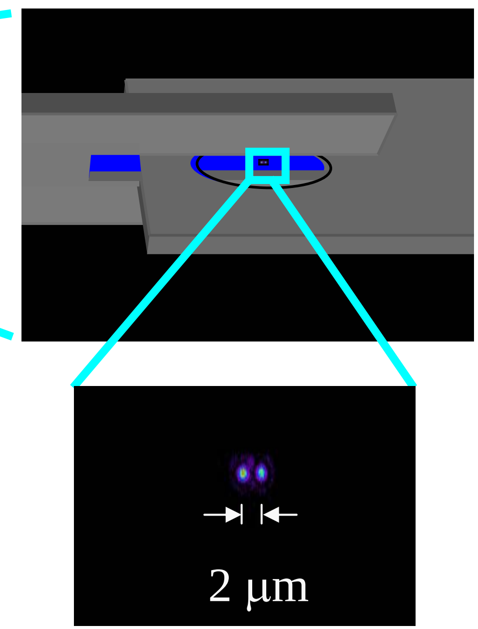


- Each ion has two long-lived electrical states representing $|0\rangle$ and $|1\rangle$
- The individual ions can be addressed by laser beams
- Means exist for initializing (optical pumping and laser cooling) and measuring the quantum state

Ion Traps



Chris Monroe, University of Michigan



Summary: State of the Art

- Quantum circuits can solve some important problems with exponentially fewer operations than classical algorithms
- Small quantum circuits have been demonstrated in the lab using various physical technologies
- Quantum cryptography has been demonstrated over long distances
- Current technologies are fragile, and appear to be limited to tens of qubits and hundreds of gates
- Big gaps remain in our understanding of quantum circuit and algorithm design, as well as the necessary implementation techniques