MULTI-CENTER CONGESTION ESTIMATION AND MINIMIZATION DURING PLACEMENT

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ABSTRACT

As technology advances, more and more issues need to be considered in the placement stage, e.g., wirelength, congestion, timing, coupling. It is very hard to consider all of them together at the same time. Thus it is good if we can optimize one cost function without affecting others. In this paper, we will study methods to optimize congestion in placement without inflicting degradations/violations in other objectives or constraint. We give a mathematical equation to predict the overflow within a region using a normal distribution approximation. According to experiments, this equation does give a good estimation of overflow. We used this equation to find the smallest regions which have enough routing resource to alleviate the congestion and propose the flexible expansion scheme in our multi-center congestion reduction (MC^2R) algorithm. Experimental results show that generally there is a correlation between the amount of reduction in congestion and the amount of change made to the placement: the more we change the placement, the more reduction in congestion we will get. However, the flexible expansion scheme is very effective in helping us reduce congestion while make only little change to the placement. Comparing to the full expansion scheme (49% congestion reduction and 6.5% change in placement), the flexible expansion scheme together with MC^2R algorithm can reduce congestion by almost the same amount (42%) with much less change made to the placement (1.8%).

1. INTRODUCTION

The physical implementation of high-performance, complex deep-submicron integrated circuits contains steps of placement and routing. Cell placement has always been a key factor for achieving designs with optimized area usage, wiring congestion and timing behavior. Traditionally, minimizing wirelength is the main objective in placement. As technology advances, the congestion problem becomes more and more important.

Congestion models routability more accurately than the wirelength objective. Usually congestion is defined by using a demand and supply relationship. A placement with zero congestion means that there is enough routing resource to supply the routing demand. Thus this placement is estimated to be routable by the given router. It is also essential to consider other performance related issue such as timing, coupling in placement.

The congestion problem in placement is not well studied. There are not many results on this problem [5, 7, 14, 15]. In [14, 15], Wang and Sarrafzadeh pointed out that the congestion objective is very ill behaved such that directly using it will not produce low congestion placement. Congestion and wirelength are globally consistent. However, in local regions, minimizing wirelength might result in high congestion. A good congestion reduction technique was obtained by using a post processing stage after the traditional wirelength minimization stage. While congestion was reduced significantly by using this technique, Wang and Sarrafzadeh also observed that the wirelength was increased by about $5-10\%$. The increase in wirelength implies that placement is changed. This change may result in violations in other performance constraint such as timing and coupling. Timing and coupling issues in placement are very hard problems [6, 13, 4, 12, 11, 2, 9, 3]. It could be very difficult and time consuming to find a placement to meet all the performance constraint. Thus we do not want the congestion reduction process to cause any trouble in this area. One way is to check the performance constraint at every step of congestion reduction process to make sure that there is no violation. However, simply doing the analysis of timing and coupling is very computationally expensive. Fortunately, a lot of performance constraint including timing and coupling are locally stable. It means that there will be no violations in the constraint as long as the change in the placement is bounded locally. Therefore in the congestion optimization problem, reducing congestion with minimal increase in wirelength will be very helpful in preventing violations in performance constraint.

In this paper, we study heuristics for congestion minimization with minimal change in placement. Since it is reported that the post processing technique is more efficient than directly minimizing congestion related objectives, we will focus on how to reduce congestion in the post processing stage while keep the increase in wirelength bounded. We assume that the placement given as the input of the post processing stage is a good placement with optimized wirelength and meet all the performance constraint as well.

We first approximate the distribution of congestion on a layout as a normal distribution. Using this approximation, we derive an equation to estimate the amount of congestion on a layout. We propose a three-step multi-center congestion reduction (MC^2R) approach to achieve the main goal of this paper, reduce congestion with minimal change in placement.

Experimental results show that this approach is extremely helpful in reducing congestion while keep the increase in wirelength bounded. Comparing to the algorithm proposed in [14] (49% congestion reduction and 6.5% change in placement), our MC^2R algorithm can reduce congestion by almost the same amount (42%) with much less change made to the placement (1.8%).

The rest of the paper is organized as follows: In Section

2, we formally define the congestion cost. Theoretical analysis on congestion distribution is described in Section 3. In Section 4, we propose our multi-center congestion reduction (MC^2R) algorithm. The experimental results are shown in Section 5 and the conclusion is in Section 6.

2. PROBLEM FORMULATION

Routing can be viewed as a demand-supply problem. All the routing layers and tracks on the chip are the supply of routing resources and wires to be routed are the routing demand. Congestion is caused by excessive wires in local regions where the routing demand exceeds routing supply. [14] used the concept of global bins to model the routing supply and demand. This model is quite reasonable and similar to what industries have been using. We will use the same model here in this paper. We mainly concentrate on placement problems when boundary of the chip is given and there is very little white space (area not occupied by cells). Thus, most nets need to be routed on the upper layers. Most present-day design follow this paradigm.

The congestion cost is defined based on the global bins. We partition a given chip area into several rectangular regions, each of these regions is called a global bin [14]. The congestion is "related" to the number of crossings between routed nets and global bin edges. Given a placement, all the cells and pins have fixed positions on the chip. We can use a global router to route all the nets. For each global bin edge, there are routed nets going across it. Therefore, for each global bin edge e, the routing demand of e, d_e, is defined as the number of the nets crossing e. The routing supply of a global edge e, s_e, is a fixed value which is a function of the length of the edge and technology parameters. A global bin edge e is congested if and only if the routing demand (number of the crossing nets) exceeds the routing supply of that edge ($d_e > s_e$). The overflow of e is defined as:

$$overflow_e = \begin{cases} \mathrm{d}_e - s_e & \text{if } d_e > s_e \\ 0 & \text{if } d_e \leq s_e \end{cases}$$

Using the above global bin and global bin edge notation, the *total overflow* of a placement is defined as the summation of the overflow for all global bin edges. The amount of total overflow is the amount of total shortage of routing resource in the placement. Thus a placement with less total overflow is less congested. Another measure of congestion could be the maximum overflow among global bin edges. It is really hard to argue which placement is more difficult to route: a placement with only one highly congested edge or a placement with a couple of slightly congested edges. It is not the purpose of this paper to discuss which is a better model for congestion. We pick the total overflow as the congestion cost in this paper. However, methods proposed here is also valid for any other reasonable congestion cost models.

3. WIRING DISTRIBUTION ON LAYOUT

The congestion map on the layout is extremely helpful to see where the congested spot is and the amount of the congestion. In this section, we will study the behavior of the wiring distribution and the relationship between this distribution and the overflow of the layout.

3.1. Wiring Distribution Can Be Approximated As a Normal Distribution

Each global bin edge has a different amount of routing demand which is expressed by the number of nets across the global bin edge. The number of nets in each global bin can be approximated as the average routing demand over all the boundary global bin edges. Normal distribution is a popular statistical function which fits a lot of natural distributions. We approximate the wiring distribution in the layout as a normal distribution because of its simplicity (more complexed models can be found in [10, 1]). Fig. 1 shows the real congestion distributions and the fitted normal distribution curves for circuit Primary2, biomed, avqs and avql. The x-axis of the figure is the number of nets at each global bin. The y-axis is the percentage/probability of global bins which has the specific number of nets in it. The actual distribution is plotted as '+' in Fig. 1. The solid curve is the fitted normal distribution function $N(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, where μ is the average value and the σ is the standard deviation. Fig. 1 shows that the actual wiring distribution can be approximated as a normal distribution.

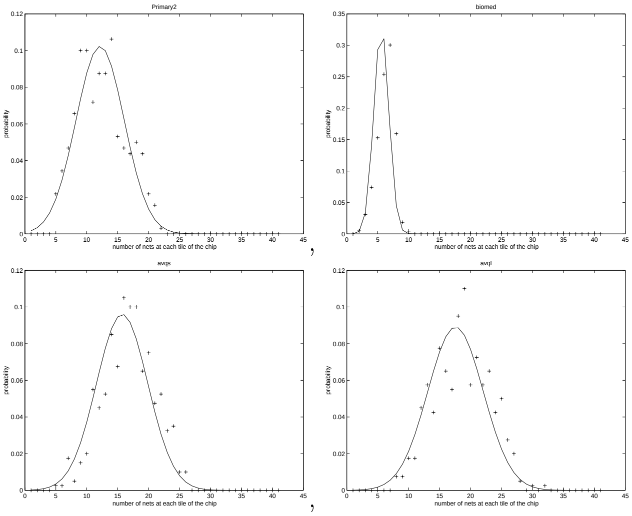


Figure 1. Wiring distribution on the layout can be approximated as a normal distribution.

3.2. Relationship Between Wiring Distribution and Overflow

If we assume that the wiring distribution among a region is a normal distribution $N(x,\mu,\sigma)$ as stated above, we can mathematically derive the amount of total overflow within this region. Let the routing supply for each unit area in this region be denoted by S and the total number of unit areas in this region be denoted by A. The overflow of this region is contributed by unit areas which has more than S nets in it. Formally, the total overflow OF can be expressed as:

$$OF = A\int_S^{\infty} \frac{1}{\sqrt{2\pi}\sigma}(x-S)e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx \qquad (1)$$

Let $t = \frac{x-\mu}{\sqrt{2}\sigma}$, then $x = \sqrt{2}\sigma t + \mu$ and $dx = \sqrt{2}\sigma dt$

From (1), we have

$$OF = A \int_{\frac{S-\mu}{\sqrt{2}\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} (\sqrt{2}\sigma t + \mu - S) e^{-t^2} dt \cdot \sqrt{2}\sigma dt \quad (2)$$

Let $S_t = \frac{S-\mu}{\sqrt{2}\sigma}$, (2) can be rewritten as

$$\frac{OF}{A} == \frac{\sigma}{\sqrt{2\pi}} \int_{S_t}^{\infty} e^{-t^2} dt^2 + \frac{\mu - S}{\sqrt{\pi}} \int_{S_t}^{\infty} e^{-t^2} dt \quad (3)$$

The integral $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2} dt$ is called the standard normal cumulative distribution function in statistics. The value of $\Phi(x)$ can be either looked up from a table in any statistics book or easily calculated by using a standard C library function. Thus we can write the second term in (3) as

$$\frac{\mu - S}{\sqrt{\pi}} \int_{S_t}^{\infty} e^{-t^2} dt = (\mu - S)\sqrt{2} \cdot (1 - \Phi(\frac{S - \mu}{\sqrt{2}\sigma})) \quad (4)$$

The integral in the first term of (3) can be directly performed. Thus we can write (3) as:

$$OF = \frac{A\sigma}{\sqrt{2\pi}} e^{-\frac{(S-\mu)^2}{2\sigma^2}} + A(\mu - S)\sqrt{2} \cdot (1 - \Phi(\frac{S - \mu}{\sqrt{2}\sigma})) \quad (5)$$

Equ. (5) is the final mathematical form to predict overflow in a region.

3.3. Experimental Validation

We will conduct experiments to test Equ.(5) in this section. Assume a layout is given and all the nets are routed. Given a region on the layout with a certain size, we get the estimated normal distribution curve for wiring, $N(x, \mu, \sigma)$, by obtaining the average and the standard deviation wiring value among all the global bin edges within this region. Then the predicted overflow value can be obtained by using (5). Finally we compare the estimated overflow value and the actual overflow value within this region.

We use circuit Primary1 and Primary2 for the test. There are in total 8×8 global bins in Primary1 and 16×20 global bins in Primary2. The size of the testing region is set to be 3×3 global bins. We select testing region at different places on the layout. Fig. 2 show the comparison between the predicted overflow and the actual overflow value for circuit Primary1 and Primary2. The x-axis is the predicted overflow value and the y-axis is the actual overflow value. If the predicted values fit the actual values well, all the data points will be close to the 45° line. Since (5) is obtained by using a statistical model, the more the number of global bins in the evaluating region, the more accurate (5) is. In our case, number of global bins is not very large. Thus it is not surprised to see the predicted overflow sometimes deviate from the actual value. However, Fig. 2 shows that Equ. (5) in Section. 3.2 does give a decent prediction in general.

4. MULTI-CENTER CONGESTION REDUCTION (MC^2R) ALGORITHMS

In [14, 15], Wang and Sarrafzadeh proposed three greedy heuristics to reduce congestion after wirelength is minimized. In all three greedy approaches, cells can be moved anywhere in the chip. This could result in violations in performance constraint. The input placement of the post processing stage has been processed so that the wirelength is minimized and the performance constraint is met. Thus it is better to limit the range of cell movement in the post processing stage. In this section, we propose a multi-center congestion reduction (MC^2R) algorithm which can let cell move within a local region.

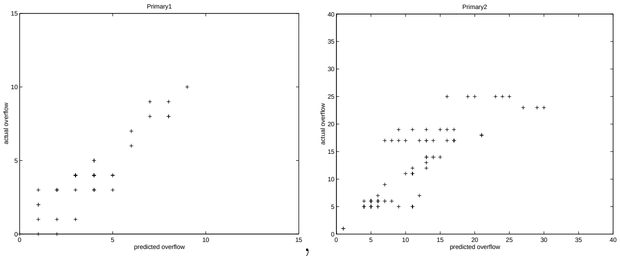


Figure 2. Comparison between the predicted overflow and the actual overflow.

Figure 3 demonstrates the flow for the general congestion reduction algorithm and the multi-center congestion reduction (MC^2R) technique. The technique consists of three major steps:

1. Identify the congested spots using the congestion map.
2. Find congestion reduction regions by using each congested spot as a center to expand.
3. Perform congestion reduction within each expanded congestion reduction region.

We will explain each step in details in each of the following subsections:

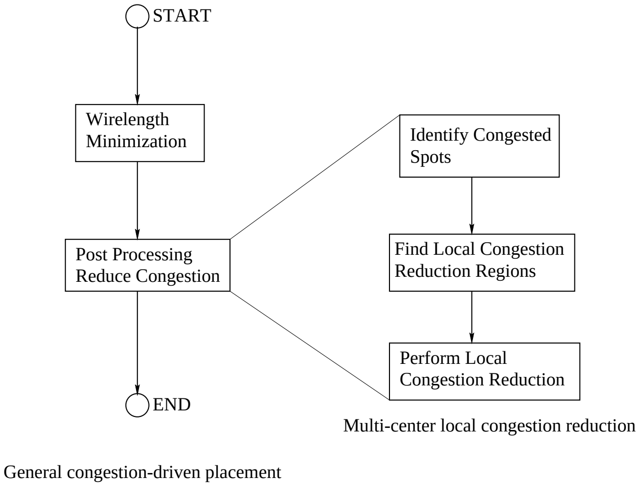


Figure 3. Work flow for congestion-driven placement and the multi-center congestion reduction technique.

4.1. Identify The Congested Spots

A global bin is congested if and only if at least one of its four global bin edges are congested. All the congested global bins in the congestion map form the core of congested spots. For the ease of shape management, we let each congested spot be in a rectangular shape. We use a minimum bounding rectangular region of a set of connected congested bins as one congested spot. The number of congested spots in the layout is equal to the number of sets of connected congested bins.

4.2. Expanding Congestion Reduction Regions

In order to limit the range of movement of cells, we need a region on the layout which contains both congested and uncongested global bins. The cell movement will be limited in this region. The congestion minimization is achieved by switching cells in different bins within this region. We call this region a *congestion reduction region.* In this paper, we let each congestion reduction region be a rectangle because rectangular regions can be easily expanded if we find out the region is not big enough to alleviate the congestion in placement.

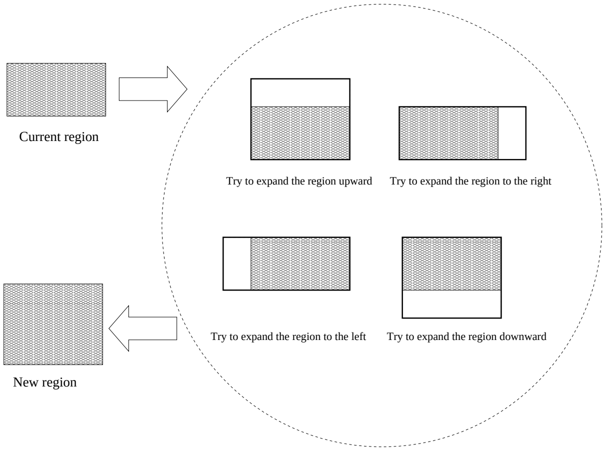
To limit the amount of change in placement, we want the size of congestion reduction regions be as small as possible. On the other hand, the routing resource supplied by this region has to be enough to meet the routing demand. Therefore, we should look for the smallest routable rectangular region around the congested spot. In order to do so, we need a way to evaluate the "routability" for a given rectangular region. We call this region an evaluating region.

Equ. (5) in Section 3.2 gives an estimated overflow for a region given A, S, μ and σ. Here A is the total number of global bin edges within the region, S is the routing supply on these global bin edges, μ is the average routing demand and σ is the standard deviation for all the routing demand values in this region. Since the total routing demand is equal to the total wirelength [14], μ can be also considered as the total wirelength in this region divided by the number of global bin edges. After the placement is changed within the region, A and S will remain the same while μ and σ will change accordingly. Since the local placement process within the evaluating region is aimed to reduce congestion, it will suppress the number of nets in congested bins and help to balance wiring density in the evaluating region. Most likely, it may sacrifice some wirelength to achieve this purpose which result in an increase in μ and a decrease in σ after the local placement.

The amount of increase in μ and the amount of decrease in σ depend on a lot of factors such as the size of the evaluating region, local net-list inside the region, initial placement outside the region, etc.. It is very difficult to accurately predict the value of μ and σ. In this paper, we use a simple empirical method to decide the amount of change in μ and σ. By observing from different experiments, we found that the amount of change in μ and σ can be approximated as a constant ratio of their original values. However, this ratio is different for the boundary and the internal global bin edges within the evaluating region. Specifically, the amount of change in μ and σ for the boundary global bin edges is smaller than the amount of change in μ and σ for the internal global bin edges. The reason for this is because that the wiring on the boundary global bin edges is affected by the initial placement outside the evaluating region more. Since the placement outside the evaluating region does not change during the MC^{2}R process, the amount of change in the boundary global bin edges is less than the amount of change in the internal global bin edges. Approximately, for boundary global bin edges, μ will remain the same and σ will decrease by 10%; for internal global bin edges, μ will increase by 2% and σ will decrease by 15%. We can know the routability after the local placement for the evaluating region by using Equ. (5) and the predicted new values for μ and σ obtained from this empirical method.

Other can estimating the routability of a evaluating region, we need an expansion scheme to find the congestion reduction region. Our expansion scheme is illustrated in Fig. 4. We expand the current evaluating region in four possible ways: up, down, left and right. For the vertical expansion (up and down), we let the current evaluating region have one more row of global bins; For the horizontal expansion (left and right), we let the current evaluating region have one more column of global bins. Then we evaluate the routability of the four newly expanded region and pick the one with the best estimated routability as the new region. We will repeat this procedure until one of the following three conditions becomes true:

1. The estimated routability of the new region is better than an expected value.
2. The new region has worse estimated routability than the original region. It means that the neighborhood is more congested than the original evaluating region.
3. The new region occupies the whole layout area so that there is no more space to expand.



Expanding process: try to expand the region in four possible ways and output the best one.

Figure 4. How to expand the current evaluating region.

We call this expansion scheme a flexible expansion scheme because the expanded region is different for different layouts. In order to compare with the flexible expansion scheme, we also propose three trivial expansion schemes: the zero expansion scheme, the constant expansion scheme and the full expansion scheme. As the name itself suggests, the zero expansion scheme will not perform any expansion. It will use the original congested spots as the congestion reduction regions. The constant expansion scheme will always expand the original congested spots by a constant amount to form the congestion reduction regions. The full expansion scheme will always use the whole layout area as the congestion reduction region. Note that the no-expansion, the constant expansion and the full expansion scheme do not require an estimation of routability. The flexible expansion scheme needs to estimate the routability of a region in order to decide the shape and the location of the congestion reduction region.

4.3. Perform Congestion Reduction

Once the congestion reduction regions are determined, we can perform a congestion optimization process within each region. In [14], Wang and Sarrafzadeh used a random greedy method and got a decent amount of reduction in congestion. For the ease of implementation, we use the same method as in [14] to perform our congestion reduction. We randomly pick a cell in the congestion reduction region and propose to move it to a randomly picked destination global bin within the same region. If the proposed move will actually result in a reduction in congestion (i.e. a reduction in the total overflow), we will make this move. Otherwise, the proposed move is cancelled and the picked cell will not be moved. This process will stop when there is no reduction in congestion after a certain number of moves are proposed.

Circuit (routing supply)	Before OF	After OF	% (ΔOF)	% (ΔWL)	% (Δ|WL|)
h2-v1(5)	4	4	0%	0%	0%
fract-v1(5)	21	18	0%	0%	0%
P1-v1(12)	58	56	3.4%	0.14%	1.01%
P1-v2(14)	21	18	14.3%	1.38%	3.01%
P2-v1(24)	26	14	46.2%	0.23%	0.52%
P2-v2(22)	74	53	28.4%	0.19%	0.77%
struct-v1(8)	10	9	10.0%	0.11%	0.43%
struct-v2(7)	41	32	21.9%	0.04%	0.50%
biomed-v1(9)	102	67	34.3%	0.09%	0.16%
biomed-v2(10)	20	15	25.0%	0.08%	0.15%
avqs-v1(25)	64	47	26.5%	0.17%	0.24%
avqs-v2(27)	26	20	23.1%	0.02%	0.13%
avql-v1(27)	81	67	17.3%	0.05%	0.08%
avql-v2(29)	37	25	32.4%	0.08%	0.12%
average	–		20.2%	0.18%	0.51%

Table 1. Multi-center congestion reduction with zero expansion scheme (v1 and v2 correspond to two different white space allocation).

5. EXPERIMENTAL RESULTS

Experiments are done to test the effectiveness of the newly proposed multi-center congestion reduction (MC^2R) algorithm in the previous section. We compare four expansion schemes explained in the previous section: zero, constant, flexible and full expansion.

For the constant expansion scheme, we always expand the original congested spot in all four directions (up, down, left, right) equally by one global bin. For instance, if the original congested spot contains $m \times n$ global bins, the new expanded region will always contain $(m + 2) \times (n + 2)$ global bins if there is enough room to expand. Since the full expansion scheme always use the whole layout area as the congestion reduction region, it is basically the same algorithm as the one proposed in [14].

We use eight MCNC standard-cell benchmark circuits for our experiments. The size of the global bin grid is chosen so that each bin has roughly 5 - 50 cells. We run two sets of experiments ('v1' and 'v2') for each circuit with different values of routing supply except h2 and fract. This is because h2 and fract are small and they are not very interesting comparing to other benchmark circuits. In practice, different routing supply values are caused by different number of routing layers or different white space alocation, etc..

The input placement to the MC^2R algorithm is a wirelength optimized placement obtained by using the NRG placement tool [8]. In this paper, we will not set actual performance constraint. We assume that the input placement meets all the performance constraint of the design. We will apply our MC^2R algorithm integrated with four expansion schemes on the input placement. Finally we will compare the amount of reduction in the overflow, the amount of change in the total wirelength and the summation of the amount of absolute change in wirelength for all nets. The amount of reduction in the overflow represents the amount of improvement in routability. The amount of change in the total wirelength represents the amount of change in the layout area to some extent. The amount of absolute change for all nets represents the amount of change in the placement.

Circuit (routing supply)	Before OF	After OF	% (ΔOF)	% (ΔWL)	% (Δ|WL|)
h2-v1(5)	4	4	0%	0%	0%
fract-v1(5)	21	21	0%	0.72%	0.72%
P1-v1(12)	58	55	5.2%	1.43%	4.86%
P1-v2(14)	21	13	33.3%	3.88%	4.32%
P2-v1(24)	26	3	88.5%	10.2%	12.1%
P2-v2(22)	74	12	83.8%	14.4%	16.2%
struct-v1(8)	10	8	20.0%	7.5%	8.0%
struct-v2(7)	41	24	41.5%	5.3%	7.5%
biomed-v1(9)	102	62	39.2%	2.76%	3.35%
biomed-v2(10)	20	8	60.0%	0.17%	0.28%
avqs-v1(25)	64	31	51.6%	2.15%	2.55%
avqs-v2(27)	26	10	61.5%	0.89%	1.12%
avql-v1(27)	81	66	18.5%	0.18%	0.36%
avql-v2(29)	37	8	78.4%	1.0%	1.3%
average	–		41.5%	3.61%	4.48%

Table 2. Multi-center congestion reduction with constant expansion scheme (v1 and v2 correspond to two different white space allocation).

Circuit (routing supply)	Before OF	After OF	% (ΔOF)	% (ΔWL)	% (Δ|WL|)
h2-v1(5)	4	4	0%	0%	0%
fract-v1(5)	21	18	14.3%	0.72%	0.72%
P1-v1(12)	58	52	10.3%	0.79%	1.86%
P1-v2(14)	21	13	33.3%	1.06%	5.13%
P2-v1(24)	26	2	92.3%	2.55%	3.38%
P2-v2(22)	74	27	63.5%	4.19%	5.43%
struct-v1(8)	10	6	40.0%	0.50%	0.68%
struct-v2(7)	41	30	24.4%	1.21%	2.56%
biomed-v1(9)	102	67	34.3%	0.50%	0.65%
biomed-v2(10)	20	9	55.0%	0.07%	0.11%
avqs-v1(25)	64	27	57.8%	1.37%	1.60%
avqs-v2(27)	26	12	53.8%	0.60%	0.70%
avql-v1(27)	81	50	38.2%	1.43%	1.74%
avql-v2(29)	37	17	54.1%	0.36%	0.42%
average	–		40.8%	1.10%	1.78%

Table 3. Multi-center congestion reduction with flexible scheme (v1 and v2 correspond to two different white space allocation).

Table 1 shows the result for the zero expansion scheme. Table 2 shows the result for the constant expansion scheme. Table 3 shows the result for the flexible expansion scheme. We also show the result for the full expansion scheme in Table 4. By comparing the zero expansion, the constant expansion and the full expansion scheme, we found that there is always a correlation between the amount of reduction in congestion and the amount of change in the placement. The more we change the placement, usually the more reduction in congestion we will get. The zero expansion has the least amount of reduction in congestion (20%) and the least amount of change in placement (0.5%). The full expansion scheme has the most amount of reduction in congestion (49%) and the most amount of change in placement (6.5%). The constant expansion scheme is in between with 42% reduction in congestion and 4.5% change in placement. Comparing to these three expansion schemes without using any routability estimation method, the flexible expansion scheme is very effective in reducing congestion while keeping placement almost unchanged. It can achieve similar amount of congestion reduction as the constant expansion scheme (41%) while only make changes to the placement less than half as much as the constant expansion scheme (1.8%).

Circuit (routing supply)	Before OF	After OF	% (ΔOF)	% (ΔWL)	% ($\Delta\lvert$WL$\rvert$)
h2-v1(5)	4	4	0%	0%	0%
fract-v1(5)	21	21	0%	2.90%	3.62%
P1-v1(12)	58	55	5.17%	2.43%	5.67%
P1-v2(14)	21	19	9.52%	14.7%	20.1%
P2-v1(24)	26	0	100%	9.96%	11.2%
P2-v2(22)	74	10	86.5%	14.3%	16.7%
struct-v1(8)	10	5	50.0%	5.52%	6.30%
struct-v2(7)	41	28	31.7%	3.24%	4.20%
biomed-v1(9)	102	38	62.7%	7.01%	8.00%
biomed-v2(10)	20	8	60.0%	2.20%	2.47%
avqs-v1(25)	64	17	73.4%	4.20%	4.77%
avqs-v2(27)	26	8	69.2%	1.71%	1.83%
avql-v1(27)	81	40	50.6%	3.80%	4.18%
avql-v2(29)	37	5	86.5%	2.17%	2.33%
average	–		48.9%	5.30%	6.53%

Table 4. Multi-center congestion reduction with full expansion scheme (v1 and v2 correspond to two different white space allocation).

6. CONCLUSION

In this paper we studied the congestion distribution on a layout. We gave a mathematical equation (9) to predict the overflow within a region using a normal distribution approximation. According to Fig. 2, Equ. (9) does give a good estimation of overflow. We used this equation to find the smallest region which have enough routing resource to alleviate the congestion and form the core of the flexible expansion scheme in our MC^2R algorithm. Experimental results show that generally there is a correlation between the amount of reduction in congestion and the amount of change made to the placement. However, the flexible expansion scheme is very effective in helping us reduce congestion while made only little change to the placement. Comparing to the full expansion scheme (49% congestion reduction and 6.5% change in placement), the flexible expansion scheme together with MC^2R algorithm can reduce congestion by almost the same amount (42%) with much less change made to the placement (1.8%).

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